EXPLORATORY FACTOR ANALYSIS OF STUDENT-TEACHERS' PERCEPTIONS OF 3D-DESCRIPTIVE GEOMETRY EDUCATION IN MOZAMBIQUE

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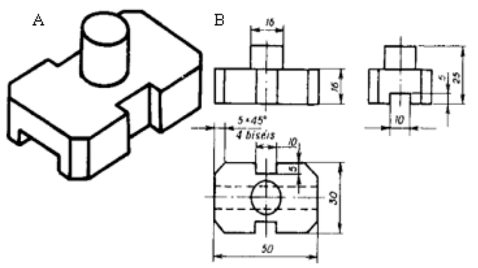
This study explores the factors underlying student-teachers' perceptions of 3D-descriptive geometry education in Mozambique. A grounded theory mixed-method approach was used to gather data from six focus groups, ten interviews and a questionnaire with 120 participants. Principal Component Analysis (PCA) for a four-factor solution was then performed, which revealed a structure with items clustered into four factors: spatial visualisation and reasoning related to 3D-geometry fundamentals; professional learning consisting of learning to teach geometry, evaluation and learning support, which encompasses items on mediating learning, and practice concerned with developing skills. Possible explanations of the findings are discussed, and their implications for further research suggested.

INTRODUCTORY OVERVIEW

This paper describes part of a study exploring student-teachers' perceptions of 3D-descriptive geometry education in Mozambique. Descriptive geometry is defined as a double projection method to study 3D geometry through 2D imaging analysis (Stachel, 2007:7), gaining insight into the structure, metrical properties, processes and principles of spatial objects, as an example of Monge's method shows in figure 1. However, the constructs involved in learning the interplay between 3D and 2D, as well as the use of intuition and reasoning, are less well understood. These elements of heuristic significance play a large part in practical 3D-descriptive geometry when a graphical method of solving space or solid-analytic problems (Slaby, 1976) is employed to enhance the global and intuitive reasoning pertaining to spatial situations (Bishop, 1980; Jones, 2002). It is believed that the ability to understand abstract geometrical concepts and offer solutions (Stachel, 2005:1) evolves throughout one's life through the development of spatial imagination (Mägi and Meister, 2002). Visualization skills for prospective professional geometry teachers in secondary schools are therefore desirable. 3D-geometry practice also helps in developing related fine motricity by means of modelling techniques (Kang et al, 2004). In the traditional teacher education curriculum in Mozambique, descriptive geometry is taught first, after which the fundamental principles of teaching in secondary schools are taught (Nhiuane et al, 2003). A curriculum review report (INDE, 2005:3-4) has indicated that some teachers are ill-prepared to teach the subject; hence leading to substandard teaching and learning. Nevertheless, neither the

Figure 1: Orthogonal projection



(Mesa & Da Costa, 2000)

report nor other Mozambican studies mention what factors lead to the successful learning of 3D-descriptive geometry, which is what this study attempts to explore.

METHOD AND PROCEDURE

This study explores the factors underlying student-teachers' perceptions of 3D-descriptive geometry in Mozambique. A grounded theory mixed-method approach (Punch, 2005:159) was used to gather data from six focus group discussions, ten interviews and a self-completed questionnaire with 120 participants selected randomly in three teacher education institutions. In this part of the study, factor analysis (See Tabachnick and Fidell, 1996) was used to condense the 40 variables measured [See appendix] relating to perceptions of 3D-descriptive geometry education down to a smaller number of meaningful underlying factors. Principal Component Analysis (PCA) for a four-factor solution was then performed, yielding an interpretable structure with items clustered into four underlying factors. Thus the factor analysis technique was deemed appropriate for this research study.

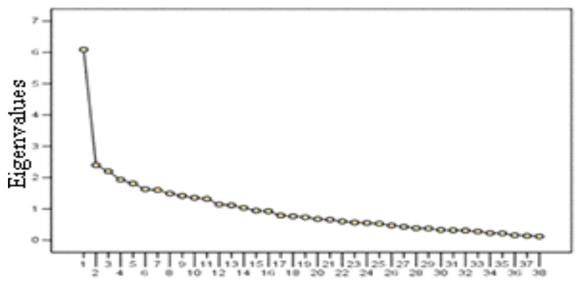
ANALYSIS AND RESULTS

It was anticipated that this part of the study would be exploratory in nature, aiming to derive the factor structure of the participants' perceptions of 3D-descriptive geometry based on the focus group and interview results. Prior to the application of factor analysis, two checks of internal reliability and validity were undertaken. Firstly, missing values were substituted with the relevant mean value. Then, the application of Cronbach's Alpha test to the 40 items yielded a score of 0.85, and thus reliability was rated as good (George and Mallery, 2003). The Kaiser-Meyer-Olkin Measure of Sampling Adequacy index was 0.58 and Bartlett's test of sphericity significantly smaller than 0.05, demonstrating that the identity matrix instrument was reliable and confirming the usefulness of factor analysis.

FACTOR STRUCTURE

At this stage of the study, exploratory factor analysis (EFA) was applied. This consisted of the principal component analysis (PCA) and orthogonal rotation performed with the 38 actual items [See appendix], as shown in figure 2. Hair et al (2006) defined factor rotation as "a process of manipulating or adjusting the factor axes to achieve a simpler and pragmatically more meaningful factor solution". The two most widely used methods are orthogonal and oblique rotation. Six criteria of the appropriateness of factor analysis for this data were examined, pertaining to: the items emerging from interview results, which were then used to design the questionnaire (Kline, 1994); the heterogeneity and sample size of questionnaire respondents (> 100) (idem.); the ratio of sample size to number of measurement

Figure 2: Orthogonal rotation scree plot



constructs (> 3:1) (Hair et al, 2006); and scree plot and rotation tests applied to questionnaire data. It was concluded that all of the necessary technical criteria were fulfilled and the results are presented below.

FACTOR ROTATION

In this study, the factor-loading criterion level of 0.35 was used (Stevens, in Field, 2000:436-37) to identify the structure of relationships among the variables. Inspection of the scree plot in figure 2 corroborates the results of PCA and table 1. This technique allowed the factors and their relative explanatory power to be identified (Cattell, in Field, 2000:437). The four factors together explain 33.2% of the variation in the data, and the sampling adequacy is 0.58, along with a significant *p*-value < .001.

Table 1: Principal Component Analysis (PCA)

Factor	*Eigenvalues*	*% of Variance*	*Cumulative % of variance*
1	6.09	16.02	16.02
2	2.39	6.31	22.33
3	2.20	5.79	28.12
4	1.94	5.08	33.19

KMO measure of sampling adequacy: 0.58; Bartlett's Test of Sphericity: 1478.222, DF = 703, Sig: .001

ORTHOGONAL ROTATION

For the orthogonal rotation techniques chosen, the method used was the factor pattern matrix (Kline, 1994). This technique is one of the simplest to interpret (Hinton et al, 2004:346). Using Varimax as the best rotation method (Kline, 1994; Hair et al, 2006), the results are given in the *appendix*. A four-factor solution from the 40 items resulted in the loading of 38 items across the 4 factors reduced to subscales ranging from 11 to 6 items. Items 32, 9, 14 and 1 were below the cutting-off point (< 0.35), and were therefore removed from the interpretation of factors below.

FACTOR INTERPRETATION OF THE FINDINGS

For a substantive interpretation of the factors, only significant loadings were considered (BMDP Statistical Software, 1993). As far as *factor one* is concerned, a close inspection of the loading order and number of items indicates that items 20, 17, 35, 33, 10 and 22 concerned the content-related and research-based learning of spatial (3D) relations, problems and solutions. They also depict visualisation abilities and graphical representation (2D). In addition, item 36 involves aspects of specificity and applications resources in learning 3D-descriptive geometry. Lower loaded items, such as 18 and 19, concern content-related knowledge about being a 3D-descriptive geometry teacher. Overall, the all-encompassing label given to this factor is **spatial visualisation and reasoning (**No. of items: 9, α = 0.73, scale mean = 4.0).

From the loading order and number of items and categories pulled together into *factor 2*, this was named becoming a geometry teacher - a dimension which loads highly (item 28 followed by 26). From the same clustering system, it was found that a dimension concerning 3D-descriptive geometry learning activities contributed to this factor with the second highest loading (as can be seen in items 25, 7 and 24). One single item (16) loaded highly to depict motivational perceptions relevant to 3D-descriptive geometry. Thus, these items which included facets of learning to teach

geometry were retained in a combined factor entitled **professional learning of 3D-descriptive geometry** (No. of items: 11, α = 0.73, scale mean = 3.9).
The items loading into *factor 3* includes a rather mixed array of participants' perceptions about teacher educators and the assessment of and support for learning geometry (containing items 12, 2, 11), 3D-geometry resources (3, 13), interaction (5 and 6) and a post-training geometry element (40). Thus, the label found to approximate the face and content validity of this factor was **evaluation and learning support** (No. of items: 8, α = 0.70, scale mean = 3.6).

The perceptions held by participants about 3D-descriptive geometry in *factor 4* were found to concern practical tasks relevant to visualisation and reasoning processes (containing items 15, 30 and 34) which loaded highly, and self-path learning and motivational perceptions of 3D-descriptive geometry teaching competencies (39 and 38) which contributed relatively high loads. Item 23, with a negative sign, was considered an anomaly item in the scale and was thus disregarded. Since the factor includes aspects of the construction-based hands-on learning of geometry, it was labelled **practice** (No. of items: 6, α = 0.73, scale mean = 4.1).

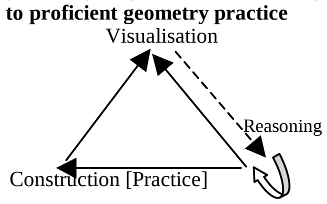
To summarise, this factor analysis has provided a clear understanding of which variables may act in concert together and how many variables have an impact in the analysis. Insofar as the working path-analytic model is concerned, subsequent research will attempt to empirically validate these perceptions.

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

The aim of this study was to investigate student-teachers' perceptions of 3D-descriptive geometry, which were clustered into four latent variables. The factor interpretation was purely intuitive, paying attention to face and content validity and following concerning research procedures.

The results have probabilistic power in developing a specific model designed to provide estimates of the effects of independent variables on dependent variables. Further analysis of the effects of a control (intervening) variable on dependent pairs will be performed using multivariate regression analysis and path analysis. A path diagram will be proposed, taking into account the effects of spatial visualisation and reasoning on practice and how it fits and explains variance within the data. Duval (1998) has described the link between spatial visualisation and reasoning (SVR) in practice as threefold, involving visualisation processes, construction processes and reasoning processes. Figure 3 shows how each of these cognitive processes can influence the others during 3D-descriptive geometry learning, and also indicates that reasoning is not always influenced by visualisation (3D to 2D or vice-versa), but can develop quite independently of other processes of construction (practice) as visualised by the circular arrow. However, visualisation might develop separately; and this study

Figure 3: Synergic links SVR leading to proficient geometry practice



(Adapted from Duval, 1998:38)

shows that SVR together does exert an influence on practice. Brisson (1992) suggested that visualisation is basic to the understanding of other components. Again, the associations between SVR and the professional learning of 3D-descriptive geometry and evaluation and learning support are worth exploring further to determine the magnitude of their impact.

The study involves a number of limitations; particularly the non-experimental nature of the study where no pre- to post-test measurement was performed. However, since the sample of the study was chosen randomly, it may also offer grounds for generalisation, however restricted, to equivalent populations. In conclusion, some implications for the field of 3D-descriptive geometry education can be drawn. Firstly, these findings can be used by policy makers and geometry curriculum bodies for appropriate curriculum renewal measures. Secondly, a new perspective and platform for practitioners' interventions in the field is offered. Finally, further refinement of the questionnaire is needed using different samples and validation procedures, since the constructs suggested require further analysis.

REFERENCES

Bishop, A.J. (1980) Spatial abilities and mathematics achievement, *Educational Studies in Mathematics*, Vol. 11, pp. 7-16.

BMDP Statistical Software Inc. (1993) *Computation with solo power analysis*. LA: BMDP Statistical Software.

Brisson, H.E. (1992) Visualization in art and science, *Leonardo, Visual Mathematics*: Special double issue, Vol. 25, No. 2(4), pp. 257-262.

Duval, R. (1998) Geometry from the cognitive point of view. *In: C Mammana and V Villani (eds.), Perspectives on the Teaching of Geometry for 21st Century: an ICMI Study*. Dordrecht: Kluwer.

Field, A. (2000) *Discovering statistics using SPSS: Advanced techniques for the beginner*. London: Sage.

George, D. and Mallery, P. (2003) *SPSS for windows: simple guide and reference 11.0 update*, 4th ed., NY: Allyn and Bacon.

Hair, J., Black, B., Babin, B., Anderson, R., & Tathan, R. (2006) *Multivariate data analysis*, 6th ed., New York: Prentice Hall.

Hinton, P.R., Brownlow, C, McMurray, I, & Cozens, B. (2004) *SPSS explained*. London: Routledge.

INDE, (2005) *Proposta de Estrutura para o Ensino Secundário Geral*. Maputo: INDE, (DOC/08/05).

Jones, K. (2002) Issues in the teaching and learning of geometry. *In: Linda Haggarty (ed.) Aspects of Teaching Secondary Mathematics: Perspectives on Practice*. Chapter 8, pp.121-139. London: Routledge.

Kang, FM., Tai, D.W.S, Ching-Lang, J. & Bin-Jiun, J. (2004) Study on development of spatial ability scale for vocational high school students, *7th UICEE Annual Conference on Engineering Education*, Mumbai-India, 3-13th February, 2004.

Kline, P. (1994) *An easy guide to factor analysis*. London, Routledge.

Mägi, R. & Meister, K. (2002) Geometry and students, engineering graphics, *BALTGRAF, Proceedings of the Sixth International Conference*, Riga, Latvia, June 13-14, 2002, 98-102.

Mesa, A. and Da Costa, D.D. (2000) *Desenho – 9ª classe*, Maputo: Diname.

Nhiuane, E.P., Samo, P. & Carrel, F.S. (2003) *Proposta curricular do curso de bacharelato e licenciatura em ensino de desenho*. Maputo: UP, (DOC/02/03).

Punch, K.F. (2005) *Introduction to social research: quantitative and qualitative approaches*, 2nd ed., London: Sage.

Slaby, S.M., (1976) *Fundamentals of three-dimensional descriptive geometry*, 2nd ed. London: John Wiley & Sons.

Stachel, H. (2007) Status of today's descriptive geometry-related education in Europe. *40th Anniversary of the Japan Society for Graphics Science*, May 12-13, University of Tokyo.

Tabachnick, B. and Fidell, L. (1996) *Using multivariate statistics*, 3rd ed., New York: Harper Collins College.

Appendix: Factor loading of scale items of the questionnaire: Rotated Component Matrix(a)

No	Items	Factors 1	2	3	4
20	Clarity and relevance of 3D-descriptive geometry content and resources	**.60**		.27	
35	Learning tasks of 3D-descriptive geometry to investigate concrete spatial problems	**.58**		.38	
17	Teaching and learning of 3D-descriptive geometry encompasses concepts and technical procedures	**.57**	.36	-.15	-.18
33	Learning spatial problems is done through the representation of 3-D objects on the drawing plane (2D)	**.53**			.16
10	Contribution to link 3D-descriptive geometry theory to practice	**.51**			.11
19	Training for understanding 3D-descriptive geometry helps to be a good teacher	**.48**	.19	-.13	.11
36	Relevant 3D-descriptive geometry bibliography as part of investigative tasks	**.47**		.30	.26
18	Three-dimensional geometry fundamentals underpin the professional knowledge of the field	**.46**	.24	-.13	
22	Improving spatial visualisation skills to engage with 3D-descriptive geometry problems	**.43**	-.15	.30	.15
32	Three-dimensional descriptive geometry learning require self-discipline, attention and hard-working	.35	.34	-.13	.14
9	Development of language abilities to communicate geometry concepts and problems	.35	.11	.19	.24
28	Getting a vision for professional growth as 3D-descriptive geometry teacher		**.58**	.15	
16	Students' interest in 3D-descriptive geometry	.12	**.56**		
25	Readiness to reflect three dimensional-descriptive problems		**.55**		.11
21	Support future 3D-descriptive geometry teacher to work with secondary school students	.17	**.46**	.44	-.15
7	Development of abilities to assess others' 3D-descriptive geometry-related views		**.45**		
26	School-based 3D-descriptive geometry practice relates to course topics	.34	**.43**	.18	-.24
27	Three-dimensional descriptive geometry training programme demands more study time	-.24	**.42**	.23	
24	Support to think of geometric problems critically	.36	**.42**		
4	Regular individual student-teachers' contact with 3D-descriptive geometry teacher educators	-.25	**.42**	.34	.28
31	Three-dimensional geometry teacher educator has a facilitative role	.12	**.42**	.22	.21
8	Development of abilities to solve spatial geometric problems\exercises and provide solutions	.22	**.42**		
14	Time for practical 3D-descriptive geometry exercises both in classroom and at home		.31		.30
1	Contribution to active and independent professional learning of 3D-descriptive geometry	.18	.24	.12	
12	Contribution to quality of 3D-descriptive geometry assignments, submission and feedback		.30	**.61**	-.20
2	Inclusion of 3D-descriptive geometry tutorials and student support	.11		**.60**	
3	Student support include 3D-geometry textbooks and books in the library	.19		**.57**	
11	Three-dimensional geometry emphasis on balance between summative and formative assessment		.19	**.54**	-.11
40	Continuous training after the 3D-descriptive geometry pre-service course is envisaged			**.52**	
13	Learning support materials include models, 3D-descriptive geometry software		.27	**.45**	
5	Regular contacts between student-teachers sharing 3D-descriptive geometry solutions and experiences		.12	**.38**	.33
6	Students understanding and reflecting on spatial reasoning experiences	.36	.11	**.38**	
15	Practice (exercise) helps understanding 3D-descriptive geometry		.15	-.18	**.61**
39	Self-study tasks of three-dimensional descriptive geometry involve trial-and-error	.13		.28	**.54**
30	Creative imagination plays role in understanding three-dimensional geometry	.18			**.50**
38	Motivational 3D-descriptive geometry tasks help good professional conduct	.12	.35	.15	**.50**
34	Models and modelling manipulation help spatial visualisation	.32		.30	**.45**
23	Perception about 3D-descriptive geometry teaching competencies	.40	.35	.21	**-.43**

Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization. Rotation converged in 18 iterations